Topology II

Exercise 1 (14.9.2017)

By doing exercises you can get bonus points for the exam as follows:
30% $\to$ 1, 40% $\to$ 2, ... , 80% $\to$ 6 points.

1. Prove, that in a metric space the intersection of finitely many open sets is open. Prove by giving a counterexample, that the result doesn't hold for arbitrary intersections.

2. Suppose, that $A \subset \mathbb{R}^2$ is open and $z \in \mathbb{R}^2 \setminus A$. Is it possible, that $A \cup \{z\}$ is open?

3. Suppose, that (X, d) is a metric space and $A \subset X$. Prove, that A is the intersection of all its' neighbourhoods.

4. Prove, that a finite metric space is discrete.

5. Let $E = C[0,1]$ and $A = \{f \in E \mid f(x) > 0 \;\; \forall\, 0 \le x \le 1\}$. Is A open, when the norm in E is (a) the sup-norm, (b) the L_1-norm? Hint: In (a) you can use the fact, that a continuous function defined in a closed interval has a maximum and minimum. In (b) consider for example the constant map $f(x) = 1$ and prove, that every neighbourhood of f contains a function $g \notin A$.

6. Suppose, that X and Y are metric spaces and $f\colon X \to Y$ is a map. Prove from the definitions, that f is continuous if and only if the inverse image of each open set $V \subset Y$ is open in X.