Complexity theory
Department of Mathematics and Statistics
Fall 2017
Exercise set 2

Exercise 1. Describe a Turing machine that adds 1 to a binary number given as input.

Exercise 2. Describe Turing machines accepting each of the following languages, as subsets of $\{0, 1, c\}^*$:

(a) $\{0, 1, c\}^*$
(b) $\emptyset$
(c) $\{\sigma \in \{0, 1, c\}^* \mid \sigma \text{ contains exactly one } c\}$
(d) $\{011\}$
(e) $\{\sigma c^n \sigma \mid \sigma \in \{0, 1\}^*, n > 0\}$.

In each case give the set of rules of the Turing machine and a brief description of how it is supposed to work.

Exercise 3. Describe a 1-tape Turing machine that solves the PALINDROME problem (over the alphabet $\{0, 1\}$), i.e., calculates the Boolean function $PAL(x)$, where $PAL(x) = 1$ iff x is a palindrome. How many moves does the machine require for an input of length n?

Exercise 4. Given a (1-tape) Turing machine M with halting states q_{halt}, q_{accept} and q_{reject}, describe a (1-tape) Turing machine M' with only one halting state q_{halt} that simulates M such for each calculation on some input

- M' halts if and only if M halts,
- if M ends in state q_{accept} then M' halts with a 1 as its first symbol on the tape,
- if M ends in state q_{reject} then M' halts with a 0 as its first symbol on the tape,
- if M ends in state q_{halt} then M' halts with neither a 0 or a 1 as its first symbol on the tape.

If M halts in $F(n)$ steps for an input of length n, how many steps does M' need for the simulating calculation?