Complexity theory
Department of Mathematics and Statistics
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Exercise set 3

Read chapters 1.1–1.3 of the book.

Exercise 1. Assume L is a language in some alphabet $\mathcal{A}$, and M is a (1-tape) Turing machine that *decides* L (i.e., calculates the characteristic function of L) in time $T(n)$. Show that there are (1-tape) Turing machines M' and M'' such that

- M' decides $\mathcal{A}^* - L$ (i.e., the complement of L)
- M'' decides $\{\sigma \in \mathcal{A}^* : \sigma^R \in L\}$, where σ^R is σ reversed.

In what times do your M' and M'' run?

Exercise 2. For $f : \{0,1\}^* \to \{0,1\}$ and a time constructible $T : \mathbb{N} \to \mathbb{N}$, show that if f is computable in time $T(n)$ by a Turing machine M using alphabet Γ, then it is computable in time $4\log|\Gamma|T(n)$ by a Turing machine $\tilde{M}$ using the alphabet $\{0, 1, \rhd, \Box\}$. (The outline is given in the book; the exercise is to give full details.)

Exercise 3. Define a *bidirectional* Turing machine to be a Turing machine whose tapes are infinite in both directions. Show that for every $f : \{0,1\}^* \to \{0,1\}^*$ and time-constructible $T : \mathbb{N} \to \mathbb{N}$, if f is computable in time $T(n)$ by a bidirectional Turing machine M, then it is computable in time $4T(n)$ by a standard Turing machine $\tilde{M}$ (with the same number of tapes).

Exercise 4. Define a *two-dimensional* Turing machine to be a Turing machine where each of its tapes is an infinite grid, and the machine can move not only left and right, but also up and down. Show that for every (time-constructible) $T : \mathbb{N} \to \mathbb{N}$ and every Boolean function f, if f can be computed in time $T(n)$ using a two-dimensional Turing machine, then f can be computed in time $c \cdot T(n)^2$, for some positive constant c, on a standard Turing machine.

Exercise 5. Define a Turing machine M to be *oblivious* if its head movements do not depend on the input but only on the input length, i.e. for every input $x \in \mathcal{A}^*$ and $i \in \mathbb{N}$, the location of each of M's heads at the ith step is only a function of $|x|$ and i. Show that for every time-constructible $T : \mathbb{N} \to \mathbb{N}$, if a language $L \subseteq \mathcal{A}^*$ is decidable (on a k-tape Turing Machine) in time $O(T(n))$, then there is an oblivious Turing machine that decides L in time $O(T(n)^2)$. Furthermore, the oblivious machine may be chosen to have only one input tape and one work/output tape.