

REAL-VARIABLE HARMONIC ANALYSIS I
2017

1. HOMEWORK SHEET
19.9.2017

Remark The centered Hardy-Littlewood maximal function is written as Mf and the uncentered one is denoted by M_0f . The uncentered Hardy-Littlewood maximal function with cubes is denoted by M_Qf .

1.1. **Homework.** Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2$. Find $Mf(x)$.

1.2. **Homework.** Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$, $f(x) = \chi_{B(0,1)}(x)$. Find an upper bound and a lower bound for $Mf(x)$.

1.3. **Homework.** Find the constants b_k and c_k , $k = 1, 2$, for the pointwise inequalities

$$b_1M_0f(x) \leq Mf(x) \leq b_2M_0f(x),$$

$$c_1M_Qf(x) \leq Mf(x) \leq c_2M_Qf(x).$$

1.4. **Homework.** Prove that the Hardy-Littlewood maximal function $Mf(x)$ is lower semicontinuous.

1.5. **Homework.** Show that the Hardy-Littlewood maximal operator M commutes with translations and dilations.