Homework set 3

September 21, 2017

1. Prove that if λ is a limit ordinal, then $\bigcup \lambda = \lambda$.

2. Prove that a set is an ordinal number iff it is a transitive set of transitive sets.

3. The union of any set of cardinal numbers is itself a cardinal number.

4. Prove the Veblen Fixed Point Schema: Assume that the mapping $\alpha \to t_\alpha$ is a *normal* operation from Ord to Ord. Then the operation has arbitrarily large fixed points, i.e. for every ordinal β we can find an ordinal γ such that $t_\gamma = \gamma$ and $\beta \in \gamma$ or $\beta = \gamma$. (See Enderton, p. 218).

5. Prove that $cof(\lambda)$ is always a cardinal.

6. Prove that if λ is a limit ordinal, in V_κ, for κ in accessible, then λ is a limit ordinal in V.

7. Prove that if $cof(\aleph_\lambda) = cof(\lambda)$, for λ limit.

8. Prove that the function f defined from g in lemma 10.31 of Kunen, page 33, is well-defined.

9. Suppose $\kappa \leq \lambda$ and suppose further that we have computed λ^κ. What is $(\lambda^+)^\kappa$?

Claim: $(\lambda^+)^\kappa = (\lambda^+) \cdot \lambda^\kappa$.

We proved in class that the left hand side of the above equation is less than or equal to the right hand side, as follows:

$(\lambda^+)^\kappa = |^\kappa(\lambda^+)|$. If $f \in^\kappa (\lambda^+)$, i.e. $f : \kappa \to \lambda^+$, then there is $\alpha < \lambda^+$ such that $f : \kappa \to \alpha$. (This is because of the regularity of λ^+.) Thus $f \in^\kappa (\alpha)$ for some $\alpha < \lambda^+$. So $f \in \bigcup^\kappa_{\alpha<\lambda^+}(\alpha)$.

But: $|\bigcup^\kappa_{\alpha<\lambda^+}(\alpha)| \leq \lambda^+ \cdot \lambda^\kappa$.

Question: Prove this last inequality.

Question: Prove the other direction, i.e. prove that the left hand side of the equation $(\lambda^+)^\kappa = (\lambda^+) \cdot \lambda^\kappa$ is greater than or equal to the right hand side.