Topology II

Exercise 4 (5.10.2017)

1. a) Prove Proposition 3.2: Let $f: X \to Y$ be a map and $a \in X$. Then the following conditions are equivalent:
(1) f is continuous at a.
(2) If $A \subset X$ and $a \in \overline{A}$, then $f(a) \in \overline{fA}$.
b) Let $f: \mathbb{R} \to \mathbb{R}$ be the function $f(x) = \sin(1/x)$, if $x \neq 0$, and $f(0) = 0$. Give an example of a set $A \subset \mathbb{R}$, such that $f\overline{A} \not\subset \overline{fA}$.

2. Prove Proposition 4.8.

3. Determine the sets $\mathrm{cl}B^2$, $\mathrm{int}B^2$, $\mathrm{ext}B^2$ and ∂B^2 in the topology of $\mathbb{R}^2$, which is induced by the projection $\mathrm{pr}_1: \mathbb{R}^2 \to \mathbb{R}$. In the target space $\mathbb{R}$ we have the usual topology.

4. A subset A of a space X is *locally closed*, if every point $a \in A$ has a neighbourhood U, such that $A \cap U \Subset U$. Prove, that A is locally closed, if and only if A is the intersection of an open and a closed set.

5. Prove, that an injective map $f: X \to Y$ is an embedding, if and only if f induces the topology of X.

6. Let $\emptyset \neq D \subset \mathbb{R}^n$ be an open convex bounded set, which contains the origin. Prove, that the function $f: \partial D \to S^{n-1}$ defined by $f(x) = x/|x|$ is a homeomorphism.