

REAL-VARIABLE HARMONIC ANALYSIS I
2017

2. HOMEWORK SHEET
10.10.2017

2.1. **Homework.** Let \mathcal{F} be a family of balls in \mathbb{R}^n of bounded diameter. Show that for every $\epsilon > 0$ there exists a countable subcollection $\mathcal{G}_\epsilon \subset \mathcal{F}$ of pairwise disjoint balls such that

$$\bigcup_{B \in \mathcal{F}} B \subset \bigcup_{B \in \mathcal{G}_\epsilon} (3 + \epsilon)B.$$

2.2. **Homework.** Prove the Dyadic Vitali-type Covering Lemma.

2.3. **Homework.** Show that for cubes Q_1 and Q_2 from the Whitney decomposition of Ω with the property $\overline{Q_1} \cap \overline{Q_2} \neq \emptyset$ the inequalities

$$\frac{1}{4} \leq \frac{\text{diam}(Q_1)}{\text{diam}(Q_2)} \leq 4$$

holds.

2.4. **Homework.** Let Q be a cube from the Whitney decomposition of Ω . Show that there are at most $(12)^n$ cubes Q' from the Whitney decomposition such that

$$\overline{Q} \cap \overline{Q'} \neq \emptyset.$$

2.5. **Homework.** Let Q be a cube from a Whitney decomposition of a set Ω . Let $1 < \sigma < 5/4$ be given. Show that each point in Ω is contained in at most $(12)^n$ of the cubes σQ .