

# Homework set 4

October 5, 2017

1. Prove that if  $A \subseteq \kappa$  is unbounded, the  $\{\alpha \in A \cap \kappa \mid \alpha \text{ is a limit point}\}$  is a club subset of  $\kappa$ , ( $\kappa$  regular).
2. Prove that if  $f$  is a normal function mapping  $\kappa$  to  $\kappa$ , then the range of  $f$  is a club subset of  $\kappa$ , ( $\kappa$  regular).
3. For the countable case of the  $\Delta$ -lemma (see attachment called delta-lemma.pdf), the continuum hypothesis is assumed. Prove that the  $\Delta$ -lemma fails in this case if we drop the continuum hypothesis.
4. Finish the proof of lemma 9.9 in Jech: prove that every weakly compact cardinal is a strong limit cardinal.
5. Finish the proof of lemma 9.26 (i): show that the branch we construct in the proof has cardinality  $\aleph_1$ .