Homework set 5

October 5, 2017

1. Prove that if $f : \kappa \to \kappa$, then the set of all points $\alpha < \kappa$ such that $f(\xi) < \alpha$ for all $\xi < \alpha$ is club.

2. Let κ be the least inaccessible cardinal such that κ is the κ-th inaccessible. Then κ is not Mahlo. (Recall the definition of a Mahlo cardinal: κ is Mahlo if the set of all regular cardinals below κ is stationary.) Hint: use $f(\lambda) = \alpha$, where λ is the αth inaccessible. (This is problem 8.6 in Jech's book. Check that your notion of inaccessible here means regular, strong limit.)

3. Prove/review this theorem due to Solovay: Let κ be a regular uncountable cardinal. Then every stationary subset of κ is the disjoint union of κ stationary sets. (This is theorem 8.10 in Jech's book.)

4. Prove that $\omega \not\to (\omega)^{<\omega}$. Hint: For $x \in (\omega)^{<\omega}$, let $F(x) = 1$ if $|x| \in x$, and 0 otherwise. If $H \subseteq \omega$ is infinite, pick $n \in H$ and show that F is not constant on $(H)^n$. This is problem 9.13 in Jech. Note that his notation is inconsistent in the question. I.e. $(A)^n = [A]^n$.