Dependence Logic
Exercise 5

1. Show the following logical consequence

$$\vec{t_1}|\vec{t_2} \Rightarrow \exists\vec{z}(\vec{t_1} \subseteq \vec{z} \wedge \vec{t_2} \neq \vec{z} \wedge \vec{t_2} \perp \vec{z}), \tag{1}$$

where $\vec{z}$ consist of fresh variables such that $|\vec{z}| = |\vec{t_1}| = |\vec{t_2}|$.

2. Show that the converse of (1) also holds:

$$\exists\vec{z}(\vec{t_1} \subseteq \vec{z} \wedge \vec{t_2} \neq \vec{z} \wedge \vec{t_2} \perp \vec{z}) \Rightarrow \vec{t_1}|\vec{t_2}$$

3. Construct a Σ_1^1 sentence ϕ such that for all finite undirected graphs $\mathbb{G}$:

$$\mathbb{G} \models \phi \Leftrightarrow \mathbb{G} \text{ is bipartite.}$$

4. Let ϕ be as in the previous exercise. Construct an independence logic sentence ψ such that for all finite undirected graphs:

$$\mathbb{G} \models \phi \Leftrightarrow \mathbb{G} \models \psi.$$

Note that by our results ψ may contain also inclusion and exclusion atoms.

5. Let $\phi, \psi \in \Sigma_1^1$. Show that there exists $\theta \in \Sigma_1^1$ such that

$$\phi \vee \psi \equiv \theta.$$

6. Let $\phi \in \Sigma_1^1$ and x a variable. Show that there exists $\theta \in \Sigma_1^1$ such that

$$\forall x \phi \equiv \theta.$$