

Complexity theory
Department of Mathematics and Statistics
Fall 2017
Exercise set 7

Read chapters 2.3 – 2.4 of the book.

Exercise 1. VERTEX COVER is the problem, where given an undirected graph G and an integer k we should decide whether there is a set S of at most k vertices in G such that all edges of the graph has at least one end point in S . Show that VERTEX COVER is NP-complete.

Exercise 2. Show that 2SAT is in P.

Exercise 3. Give a parsimonious polynomial time Karp reduction from SAT to 3SAT. (Hint: When does the reduction we looked at not give a unique satisfying assignment for the 3CNF formula, and what can be done about it without actually checking truth values?)

A language L is polynomial-time Cook reducible to a language L' if there is a polynomial time Turing machine M that, given an oracle for deciding L' , can decide L . An oracle for L' is an extra device plugged to M that checks whether $x \in L'$ in one step (e.g. M has an extra tape, s.t. when M writes a string x on this tape and goes into a special “invocation” state, then the string gets overwritten by 1 or 0 depending on whether $x \in L'$ or not, and the overwriting happens in one single step).

Exercise 4. Show that

- (a) the notion of Cook reducibility is transitive, and
- (b) 3SAT is Cook-reducible to TAUTOLOGY, where TAUTOLOGY is the set of (codes of) Boolean formulas that are satisfied by all assignments.

Exercise 5. Show that $\overline{\text{SAT}}$ (i.e. the complement of SAT) is NP-hard under Cook reductions, i.e. every language in NP reduces to $\overline{\text{SAT}}$ via a Cook reduction.