Dependence Logic
Exercise 6

1. Construct a dependence logic sentence such for all finite structures $\mathcal{M}$:

$$\mathcal{M} \models \phi \Leftrightarrow |\text{Dom}(\mathcal{M})| \text{ is even.}$$

2. Let ϕ be a FO($\perp_c$)-sentence with an infinite model or arbitrarily large finite models. Show that then ϕ has models in all infinite cardinalities. (Hint: use a similar argument as in Theorem 45 of the lecture notes. In particular, you may assume that first-order sentences satisfy the claim.)

3. Let $\phi := \vec{t}_2 | \vec{t}_2$ and V a finite set of variables that contains the free variables of ϕ. Define $\tau_{\phi,V}$ and prove the claim of Theorem 43 in this case.

4. Let $\phi := \psi \vee \theta$ and V a finite set of variables that contains the free variables of ϕ. Show that the translation $\tau_{\phi,V}$ (defined in the lecture notes) satisfies the claim of Theorem 43 assuming the translations of ψ and θ satisfy it.

5. Show that for every formula ϕ of dependence logic there is a formula $\psi \equiv \phi$ in so-called *prenex normal form* of the form

$$\psi := Qx_1 \ldots Q_n \psi,$$

where $Q_i \in \{\exists, \forall\}$ and ψ is a quantifier-free formula. (Hint: Use the equivalences of Proposition 29 of the lecture notes and renaming of bound variables.)