Complexity theory
Department of Mathematics and Statistics
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Exercise set 8

Read chapters 2.4 – 2.7 of the book.

Exercise 1. Show that **coNP** is closed under union and intersection, i.e., if $L_1, L_2 \in$**coNP** then so are $L_1 \cup L_2$ and $L_1 \cap L_2$.

Exercise 2. Show that **NP**=**coNP** iff `3SAT` and `TAUTOLOGY` are polynomial-time reducible to one another.

Exercise 3. Give a definition of **NEXP** without using nondeterministic Turing machines, analogous to our definition of the class **NP**, and prove that the definition is equivalent to the one using nondeterminism.

Exercise 4. Suppose $L_1, L_2 \in$**NP**$\cap$**coNP**. Then show that $L_1 \oplus L_2$ is in **NP**$\cap$**coNP** where $L_1 \oplus L_2 = \{x : x \text{ is in exactly one of } L_1, L_2\}$.

Exercise 5. Let R be a relation that is *polynomially bounded*, i.e., there exists a polynomial p such that if $(x, w) \in R$ then $|w| \leq p(|x|)$, and *polynomially verifiable*, i.e. $R \in$**P**. Then R defines the **NP**-language

$$L_R := \{x \mid \exists w : (x, w) \in R\}.$$

The *search problem* over R is: given x, find a w for which $(x, w) \in R$, if such a w exists. The *decision problem* over R is: given x, determine whether $x \in L_R$. A relation is *self-reducible* if the search problem over R is Cook-reducible to the decision problem over R. (Cook-reductions were defined in last week's exercises.)

Show that all (polynomially bounded and polynomially verifiable) relations whose decision problems are **NP**-complete, are self-reducible.

(It is conjectured that not all languages in **NP** are self-reducible. An example might be the natural relation corresponding to the factoring problem: deciding whether a number is composite is in **P**, but factoring composite numbers in polynomial time is not believed possible.)