Topology II

Exercise 7 (2.11.2017)

1. Proposition 8.7 (3). Prove, that the composite of two identification maps is an identification map.

2. Suppose, that $f\colon X \to Y$ and $g\colon Y \to X$ are continuous and $g \circ f = \mathrm{id}$. Prove, that f is an embedding and g is an identification map.

3. Let A be the closed interval $[0, 1]$. Prove, that $\mathbb{R}/A \approx \mathbb{R}$.

4. Prove: If $A \subset X$ and if $\{A\} \Subset X/A$, then $A \Subset X$. Deduce, that the space $\mathbb{R}/]0, 1[$ is not a Hausdorff space, and thus not homeomorphic with $\mathbb{R}$.

5. Let R be an equivalence relation in a space X, and let S be an equivalence relation in a space Y. Suppose, that f is a map, which satisfies: $xRx' \Rightarrow f(x)Sf(x')$ for all $x, x' \in X$. Prove, that there exists a unique map $\hat{f}\colon X/R \to Y/S$, for which $\hat{f} \circ p_R = p_S \circ f$. Prove, that $\hat{f}$ is continuous, if f is continuous.

6. Suppose, that X and Y are spaces and $X \cap Y = \emptyset$. Suppose, that $A \Subset X$ and $f\colon A \to Y$ is continuous. Denote $W = X \cup Y$ and $Z = W/R$, where Z has the disjoint union topology, and R is the equivalence relation, whose classes are the singleton sets $\{x\}$, where $x \in X \setminus A$, and the sets $f^{-1}\{y\} \cup \{y\}$, where $y \in Y$. The space Z is the so called *adjunction space*, which is obtained by "glueing X to Y using the map $f\colon A \to Y$". Denote $p\colon W \to Z$ the projection. Prove, that $p|Y$ is an embedding and $pY \Subset Z$.