Complexity theory
Department of Mathematics and Statistics
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Exercise set 9

Read chapters 3.1 – 3.2 of the book.

Exercise 1. Show that there exists a *nondeterministic universal Turing machine* $\mathcal{NU}$ that simulates any nondeterministic Turing machine with constant overhead, i.e., for every string α and input x, $\mathcal{NU}(x, \alpha) = M_\alpha(x)$ and if M_α halts in time T on input x, then $\mathcal{NU}$ will halt in time CT on input x, α, where C depends only on the simulated machine M_α. Hint: use guesses also in the simulation to avoid the logarithmic overhead.

Exercise 2. Show that if $\mathbf{DTIME}(n)$=$\mathbf{NTIME}(n)$, then $\mathbf{P}$=$\mathbf{NP}$.

Exercise 3. Show that $\mathbf{coNP} \subseteq \mathbf{EXP} \subseteq \mathbf{coNEXP}$. What do we know about the inclusions?

Exercise 4. Show that the following language is undecidable:

$$\{\llcorner M \lrcorner : \ M \text{ is a machine that runs in } 100n^2 + 200 \text{ time}\}.$$

Exercise 5. Say that a class C_1 is *superior to* a class C_2 if there is a machine M_1 in class C_1 such that for every machine M_2 in class C_2 and every large enough n, there is an input of size between n and n^2 on which M_1 and M_2 answer differently.

(a) Is $\mathbf{DTIME}(n^{1.1})$ superior to $\mathbf{DTIME}(n)$?
(b) Why does our proof of the Nondeterministic Time Hierarchy Theorem not prove that $\mathbf{NTIME}(n^{1.1})$ is superior to $\mathbf{NTIME}(n)$?