Problem set: Infinite games

1 Problems

Problem 1: Let $A = \{(n_0, n_1, ...) \in \omega^\omega : n_{2i+1} = n_{2i}^2\}$. Which player has a winning strategy?

Problem 2: Let $A = \{(n_0, n_1, ...) \in \omega^\omega : n_0 = 2, n_{2i+2} = n_{2i+1}^2\}$. Which player has a winning strategy?

Problem 3: Fix $i \in \omega$. Let $A = \{(n_0, n_1, ...) \in \omega^\omega : n_i = 0\}$. Which player has a winning strategy?

Problem 4: Let $A \subseteq \omega^\omega$ be countable. Which player has a winning strategy?

Problem 5: Let $A = \{(n_0, n_1, ...) \in \omega^\omega : n_i = 0 \text{ for all prime } i\}$. Which player has a winning strategy?

Problem 6: Prove rigorously that always one of the players does **not** have a winning strategy in $G(A)$.

Solution: Case 1: Player I has a winning strategy f in $G(A)$

Problem 8: Suppose $f : \omega \to \omega$ is a bijection. If $A \subseteq \omega^\omega$, let $A' = \{(f(n_0), f(n_1), ...) : (n_0, n_1, ...) \in A\}$. Show that $G(A)$ is determined if and only if $G(A')$ is determined.

Solution: If $s = (n_0, n_1, ...)$ is a play in $G(A)$, then, of course, $s' = (f(n_0), f(n_1), ...)$ is a play in $G(A')$. If player I won s, then he won s' and vice versa. If player II won s, then she won s' and vice versa. There is an obvious translation of strategies between the games $G(A)$ and $G(A')$ so that one is a winning strategy of a particular player in $G(A)$ if and only if its translation is a winning strategy of the same player in $G(A')$.