

Complexity theory
Department of Mathematics and Statistics
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Exercise set 10

Read chapters 3.3 – 3.4 of the book and discuss with a friend/relative/aquaintance what the Baker-Gill-Solovay theorem says about the P vs. NP question.

Exercise 1. Show that the function H defined in the proof of Ladner's theorem is computable in polynomial time.

Exercise 2. Show that $\text{coNP} \subseteq \mathbf{P}^{\text{NP}}$, where $\mathbf{P}^{\text{NP}} = \bigcup_{A \in \text{NP}} \mathbf{P}^A$.

Exercise 3.

- (a) Modify the proof of the Baker-Gill-Solovay theorem to show that there is $B \in \text{EXP}$ such that $\text{NP}^B \neq \mathbf{P}^B$.
- (b) Are there oracles B such that if $\mathbf{P}^B \neq \text{NP}^B$ then actually $\mathbf{P} \neq \text{NP}$?

Exercise 4. Let M be a deterministic Turing machine that only queries oracle strings that are shorter than the input string. Show that if $A = L(M^A)$ and $B = L(M^B)$ (where $L(M)$ is the language decided by M) then $A = B$. Hint: induction.

Exercise 5. Show that there is an oracle A and a language $L \in \text{NP}^A$ such that L is not polynomial-time reducible to 3SAT even when the machine computing the reduction is allowed access to A .

Exercise 6. In the lecture on Tuesday we noted that $\text{NP} \subseteq \mathbf{P}^{\text{SAT}}$.

- (a) Work out a natural definition of adding oracles to oracle machines and show that $(M^A)^B = (M^B)^A$ and $(M^A)^A = M^A$.
- (b) Deduce that $(\mathbf{P}^{\text{SAT}})^{\text{SAT}} = \mathbf{P}^{\text{SAT}}$.
- (c) Why cannot we deduce $\text{NP}^{\text{SAT}} \subseteq \mathbf{P}^{\text{SAT}}$ from the above?