Topology II

Exercise 10 (23.11.2017)

1. Complete the proof of Example 11.11.1 by proving, that $\mathcal{B}$ is a basis for some topology, and the space is Hausdorff.

2. Suppose, that $f, g\colon X \to Y$ are continuous, X and Y are Hausdorff, $A \subset X$ is dense and $f|A = g|A$. Prove, that $f = g$.
Does this result hold without the assumption that X is Hausdorff? Does it hold without the assumption that Y is Hausdorff?

3. Prove, that the properties N_1 and N_2 are hereditary.

4. Prove, that a closed subspace of a Lindelöf space is Lindelöf.

5. Let X and A be as in Example 11.11.1. Prove:
(a) X is separable.
(b) The subspace A is not separable.
(c) X is not N_2.
(d) X is not Lindelöf.
(e) Is X N_1?
(f) Is X metrizable?

6. Let $E = C[0, 1]$, the set of continuous maps $f\colon I \to \mathbb{R}$, equipped with the sup-norm. Prove, that E is separable, and hence N_2. Hint: Divide I into subintervals of equal length and consider functions, which obtain rational values at the end points of the subintervals, and which are affine in the subintervals, that is, of the form $f(t) = at + b$.