Complexity theory
Department of Mathematics and Statistics
Fall 2017
Exercise set 14: practice for exam

Read chapters 0-5 of the book for the exam.

Exercise 1. What is the definition or statement and main idea of proof of the following:

- decision problem
- Boolean formula
- `3SAT`
- `PATH`
- `TQBF`
- `TAUTOLOGY`
- `VERTEX COVER`
- `3COL`
- `GRAPH_ ISOM`
- `INDSET`
- `dHAMPATH`
- What is known of the complexity of the above problems? How is it proved?
- configuration
- verifier
- **DTIME**(T(n)), **NTIME**(T(n))
- **SPACE**(S(n)), **NSPACE**(S(n))
- **P**, **NP**, **L**, **NL**, **EXP**, **PSPACE**, **PH**,
- **NP**-hard, **NP**-complete
- Ladner's theorem
- Time hierarchy theorem
- Space hierarchy theorem
- Savitch's theorem
- Immerman–Szelepcsenyi theorem
- $\leq_p$ and $\leq_l$

Exercise 2. Explain the error in the following proof of $\mathbf{P} \neq \mathbf{NP}$:

(a) Suppose **P**=**NP**.
(b) Then for some $k \in \mathbb{N}$, `SAT` $\in \mathbf{DTIME}(n^k)$.
(c) As every language in **NP** is reducible to `SAT`, $\mathbf{NP} \subseteq \mathbf{DTIME}(n^k)$.
(d) Due to the assumption **P**=**NP**, $\mathbf{P} \subseteq \mathbf{DTIME}(n^k)$.
(e) Then $\mathbf{DTIME}(n^k) \subsetneq \mathbf{DTIME}(n^{k+1})$ is a contradiction with $\mathbf{P} \subseteq \mathbf{DTIME}(n^k)$.

Exercise 3. Give a Turing machine that decides the language

(a) $\{0001\}$,

(b) $\{(0001)^n \in \{0,1\}^* : n \in \mathbb{N}\}$, i.e. the language consists of sequences that are (finite) repetitions of the string '0001' (e.g. 0001, 00010001)

What can you say about the complexity of the language based on your Turing machine.

Exercise 4. Show that the classes **P**, **NP**, **L**, **NL** are closed under logspace reductions. What about polynomial time Karp reductions?

Exercise 5. What would be the consequences if $\texttt{TAUTOLOGY} \leq_p \texttt{3COL}$?

Exercise 6. Show that $\mathbf{P}^A = \mathbf{NP}^A$ for all **PSPACE**-complete languages A.

Exercise 7. Show that if $\mathbf{SPACE}((\log n)^2) \subseteq \mathbf{P}$, then $\mathbf{L} \neq \mathbf{P}$.