HOMEWORK 1 (EC and MF)

(1) (10pts) Consider $V \subset \mathbb{A}^2(\mathbb{C})$ defined by $y^2 = x^3$.
   (a) Find all singular points of V.
   (b) Find generators of the maximal ideal M_P at $P = (0, 0)$. (see e.g., Silverman Page 5 for definition). Calculate the dimension of the $\mathbb{C}$-vector space M_P/M_P^2.

(2) (10pts) Consider $V \subset \mathbb{A}^2(\mathbb{C})$ defined by $4x^2y^2 = (x^2 + y^2)^3$. Find all singular points.

(3) (10pts) Consider the projective space $\mathbb{P}^n(\mathbb{C})$, and consider the Galois group of $\mathbb{C}$ over $\mathbb{R}$ (which is generated by conjugation). Show that the fixed point of $\mathbb{P}^n(\mathbb{C})$ under the group action is precisely $\mathbb{P}^n(\mathbb{R})$. (hint: consider $n = 1$ first.)

(4) (10pts) Consider $V : y^2 = x^3+17$. Let $P_1 = (-1, 4)$ and $P_2 = (2, 5)$, and let L be the line through P_1, P_2. The line intersects with V at 3 points. Find the other intersection point P_3. (one can see that $P_3 \in V(\mathbb{Q})$, and this is a general phenomenon for elliptic curves. See exercise I.5 on Silverman, page 15)