

Computability theory

Exercise 1

Show that the following functions are URM-computable.

1. $f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x \neq 0 \end{cases}$

2. $f(x) = 5$

3. $f(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$

4. $f(x) = \begin{cases} x/3, & \text{if } x \text{ is a multiple of } 3 \\ \text{undefined,} & \text{otherwise} \end{cases}$

5. Suppose that P is a program without any jump instructions. Show that there exists a number m such that, for all x , either

$$\begin{aligned} f_P^{(1)}(x) &= m, \text{ or} \\ f_P^{(1)}(x) &= x + m. \end{aligned}$$