

# Computability theory

## Exercise 3

1. Show that the following functions are computable:

1.  $\lfloor \sqrt{x} \rfloor$ ,
2.  $\text{HFC}(x, y)$  = the highest common factor of  $x$  and  $y$ ,

2. Show that the following functions are computable:

1.  $f(x)$  = the number of prime divisors of  $x$ ,
2.  $\phi(x)$  = the number of  $y < x$  such that  $\text{HFC}(x, y) = 1$ ,

3. Define  $\pi(x, y) = 2^x(2y + 1) - 1$ .

1. Show that  $\pi: \mathbb{N}^2 \rightarrow \mathbb{N}$  is a bijection,
2. Show that the functions  $\pi_1$  and  $\pi_2$  satisfying for all  $z \in \mathbb{N}$

$$\pi(\pi_1(z), \pi_2(z)) = z,$$

are also computable.

4. Let  $f(x)$  be defined by setting  $f(0) = 1$ ,  $f(1) = 1$ , and  $f(x + 2) = f(x) + f(x + 1)$ . Show that  $f$  is computable. (Hint: Show first that  $g(x) = 2^{f(x)}3^{f(x+1)}$  is computable.)

5. Show that the predicate 'x is power of a prime number' is decidable.