HOMEWORK 3 (EC and MF)

(1) (10pts) Consider the curve defined by

$$(EQ1): \quad y^2 + xy = x^3 - \frac{36}{j_0 - 1728}x - \frac{1}{j_0 - 1728}$$

where $j_0 \in \mathbb{C}, j_0 \neq 0, 1728$.

(a) Use the substitution $y \mapsto \frac{1}{2}(y - a_1x - a_3)$ (see [Sil, P42]) to transform (EQ1) to

$$(EQ2): \quad y^2 = 4x^3 + b_2x^2 + 2b_4x + b_6$$

(b) Use the substitution $(x, y) \mapsto (\frac{x-3b_2}{36}, \frac{y}{108})$ (see [Sil, P43]), to transform (EQ2) to

$$(EQ3): \quad y^2 = x^3 - 27c_4x - 54c_6$$

(c) Use (EQ3) to calculate the Δ and j-invariant.

(2) (10pts) Consider the curve defined by

$$(EQ1): \quad y^2 = x(x-1)(x-\lambda)$$

where $\lambda \in \mathbb{C}, \lambda \neq 0, 1$.

(a) Use the substitution $y \mapsto \frac{1}{2}(y - a_1x - a_3)$ to transform (EQ1) to

$$(EQ2): \quad y^2 = 4x^3 + b_2x^2 + 2b_4x + b_6$$

(b) Use the substitution $(x, y) \mapsto (\frac{x-3b_2}{36}, \frac{y}{108})$, to transform (EQ2) to

$$(EQ3): \quad y^2 = x^3 - 27c_4x - 54c_6$$

(c) Use (EQ3) to calculate the Δ and j-invariant.

(3) (10pts) Consider meromorphic function $f(z)$ which has a zero at a of order m. Compute the residue of $\frac{zf'(z)}{f(z)}$ at a.