HOMEWORK 4 (EC and MF)

(1) (15pts) Let $\Lambda = \mathbb{Z}i \oplus \mathbb{Z}1 \subset \mathbb{C}$ be a lattice, and let $\wp(z)$ be the Weierstrass function defined by Λ (interesting fact: in LaTex, there is a special command \wp to denote "Weierstrass p".). Show that:
  (a) $\wp(iz) = -\wp(z)$
  (b) $\wp(\bar{z}) = \overline{\wp(z)}$ (here $\bar{*}$ is conjugation.)
  (c) $\wp(z)$ has $\frac{1+i}{2}$ as a zero with multiplicity 2.

(2) (10pts) If f, f' are modular forms of weight $2k, 2k'$ respectively, show that $f \cdot f'$ is a modular form of weight $2k + 2k'$. If f is a cusp form, then so is $f \cdot f'$.

(3) (10pts) If $\Gamma \subset SL_2(\mathbb{Z})$ is a *congruence subgroup* (see [DS, page 13, Def. 1.2.1] for the definition), and $\gamma \in SL_2(\mathbb{Z})$, show that $\gamma^{-1}\Gamma\gamma$ is also a congruence subgroup.