

HOMEWORK 5 (EC and MF)

- (1) (10pts) Let  $m, d \in \mathbb{Z}^{>0}$ , let

$$f := \sum_{b=1}^d e^{2\pi i \frac{bm}{d}}.$$

Then

- if  $d$  divides  $m$ , then  $f = d$ .
- if  $d$  does not divide  $m$ , then  $f = 0$ .

- (2) (20pts) Let  $G_k$  be the weight  $2k$  Eisenstein series.

- Find all zeros of  $G_4$  in  $\mathcal{H}/SL_2(\mathbb{Z})$  (hint: use Thm 4 on [Serre, Page85].)
- Find all zeros of  $G_5$  in  $\mathcal{H}/SL_2(\mathbb{Z})$
- Find all zeros of  $G_7$  in  $\mathcal{H}/SL_2(\mathbb{Z})$
- Show that  $G_6$  has a unique *simple* zero in  $\mathcal{H}/SL_2(\mathbb{Z})$ . (this one is very challenging! The proof that I can come up with uses results in [Serre, Section 4: expansion at infinity]. You can certainly write down whatever you can obtain.)