Computability theory

Exercise 6

1. Show that every computable unary function f has infinitely many indices (i.e., numbers e such that $f = \phi_e$).

2. Let $f(x, y)$ be a total computable function. For each m, let g_m be the computable function given by

$$g_m(y) = f(m, y).$$

Construct a total computable function h such that for each m, $h \neq g_m$.

3. Let $f(x)$ be a partial unary computable function and $m \in \mathbb{N}$. Construct a non-computable function g such that $g(x) \simeq f(x)$ for $x \leq m$.

4. Show that there is a total computable function k such that for each n $k(n)$ is an index of the function $\lfloor \sqrt[n]{x} \rfloor$.

5. Let $n \geq 1$. Show that there is a total computable function s such that

$$W^{(n)}_{s(x)} = \{(y_1, \ldots, y_n) \in \mathbb{N}^n \mid y_1 + y_2 + \cdots + y_n = x\}.$$