# Computability theory

## Exercise 7 (16.3)

**1.** Show (a sketch of a proof) that the function $s_n^m$ defined in The s-m-n theorem are primitive recursive.

**2.** Show that there is a decidable predicate $Q(x, y, z)$ such that

- $y \in E_x$ if and only if $\exists z Q(x, y, z)$,
- if $y \in E_x$ and $Q(x, y, z)$, then $\phi_x((z)_1) = y$.

**3.** Deduce that there is a computable function $g(x, y)$ such that

- $g(x, y)$ is defined if and only if $y \in E_x$,
- if $y \in E_x$, then $g(x, y) \in W_x$ and $\phi_x(g(x, y)) = y$; i.e., $g(x, y) \in \phi_x^{-1}(\{y\})$.

**4.** Deduce that if $f$ is an injective computable function, then $f^{-1}$ is also computable.