Computability theory

Exercise 10

1. Show that the set $\{x \mid \phi_x \text{ is not injective}\}$ is r.e.

2. Show that there are total computable functions k and l such that for every x, $W_x = E_{k(x)}$ and $E_x = W_{l(x)}$.

3. Let $A \subseteq \mathbb{N}$ be an r.e. set. Show that the sets $\bigcup_{x \in A} W_x$ and $\bigcup_{x \in A} E_x$ are also r.e. Show also that $\bigcap_{x \in A} W_x$ is not always r.e. by considering sets of the form

$$K_t = \{x \mid P_x(x) \text{ does not halt in } t \text{ steps}\}.$$

4. Let f be a unary computable function and $A \subseteq \mathrm{Dom}(f)$. Show that $g = f \restriction A$ is computable if and only if A is r.e.

5. Let A be an infinite r.e. set. Construct a total computable injective function f such that $A = \mathrm{Ran}(f)$ (in other words, f enumerates A without repetitions).

6. Let f be a total computable function and $A, B \subseteq \mathbb{N}$ such that A is recursive and B is r.e. Show that $f^{-1}(A)$ is recursive and the sets $f(A)$, $f(B)$, and $f^{-1}(B)$ are r.e. What extra information do we get if f is also a bijection?