Computability theory
Exercise 11

1. Prove Rice's theorem from the Rice-Shapiro Theorem.

2. Let $K_i = \{x \mid \phi_x(x) = i\}$ for $i \in \{0, 1\}$. Show that K_0 and K_1 are both r.e. and that they are recursively inseparable, that is, $K_0 \cap K_1 = \emptyset$ and there is no recursive set C such that $K_0 \subseteq C$ and $K_1 \subseteq \mathbb{N} \setminus C$.

3. Show that the set $\{x \mid \phi_x \text{ is injective}\}$ is productive.

4. Let $A, B \subseteq \mathbb{N}$ be such that B is r.e. and $A \cap B$ is productive. Show that then A is also productive.

5. Let $\mathcal{A}$ be a set of unary computable functions. Assume that $g \in \mathcal{A}$ is such that $\theta \notin \mathcal{A}$ for any finite $\theta \subseteq g$. Show that the set $\{x \mid \phi_x \in \mathcal{A}\}$ is productive. Hint: Follow the proof of the Rice-Shapiro theorem.

6. Show that the set $\{x \mid \phi_x \text{ is total}\}$ is productive.