Introduction to Quantum Computation
Master's Programme in Mathematics and Statistics
Fall 2018
Exercise set 2

Exercise 1. Which of the following are possible states of a qubit?

(a) $\frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$
(b) $\cos\varphi|0\rangle + i\sin\varphi|1\rangle$
(c) $\frac{i}{\sqrt{3}}|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}|1\rangle$
(d) $\frac{1}{2}|0\rangle - \frac{i}{2}|0\rangle - \frac{1}{2}|1\rangle + \frac{i}{2}|1\rangle$
(e)
$$\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}|0\rangle$$

Exercise 2. Show that the definition of projector is independent of the choice of orthonormal basis.

Exercise 3. Show that a projector is

(a) a projection,
(b) self-adjoint.

Exercise 4.

(a) $(AB)^\dagger = B^\dagger A^\dagger$,
(b) interpreting c as the operator 'scalar multiplication with c', $c^\dagger = c^*$,
(c) $(A^\dagger)^\dagger = A$,
(d) denoting $|v\rangle^\dagger := \langle v|$, we have $(A|v\rangle)^\dagger = \langle v|A^\dagger$, and thus $(A|v\rangle, B|w\rangle) = \langle v|A^\dagger B|w\rangle$,
(e) $(|w\rangle\langle v|)^\dagger = |w\rangle\langle w|$.

Exercise 5. An operator U satisfies $U^\dagger U = I$ if and only if it is a bijection that preserves inner products.

Exercise 6. What are the probabilities if the state $\psi = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$ is measured with respect to the basis $\{|+\rangle, |-\rangle\} := \{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$? Can you give an observable corresponding to the measurement?