

Finite model theory  
Problems 2  
Tuesday 18.9.2018

1. Let  $\tau$  be a finite vocabulary without function symbols. Determine the number of atomic  $\tau$ -sentences without variables.
2. Let  $\{E\}$  be a binary relation symbol. Construct a first-order sentence  $\varphi$  such that

$$\mathfrak{A} \models \varphi \Leftrightarrow E^{\mathfrak{A}} \text{ is an equivalence relation.}$$

3. Let  $\tau$  be a finite vocabulary without function symbols, and  $\mathfrak{A}$  a finite  $\tau$ -model. Show that there exists a first-order sentence  $\varphi_{\mathfrak{A}}$  such that for all  $\mathfrak{B}$  the following holds:

$$\mathfrak{B} \models \varphi_{\mathfrak{A}} \Leftrightarrow \mathfrak{B} \cong \mathfrak{A}.$$

4. Let  $f$  be a unary function symbol. Construct a non-contradictory  $\{f\}$ -sentence  $\varphi$  such that the following holds for all  $\mathfrak{A}$  of vocabulary  $\{f\}$ :

$$\mathfrak{A} \models \varphi \Rightarrow \text{Dom}(\mathfrak{A}) \text{ is infinite.}$$

5. Let  $\mathfrak{A}_n$  be a structure of vocabulary  $\{+\}$  (binary function symbol) such that  $\text{Dom}(\mathfrak{A}_n) = \{0, \dots, n-1\}$  and  $+$  is interpreted as addition modulo  $n$ . Give an example of a sentence  $\varphi$  such that  $\mathfrak{A}_{13} \models \varphi$  but  $\mathfrak{A}_{17} \not\models \varphi$ .