

Finite model theory
 Problems 3
 Tuesday 25.9.2018

1. Show that for every $\mathcal{L}_{\infty,\omega}$ -sentence φ over a finite τ there is a $\mathcal{L}_{\omega_1,\omega}$ -sentence ψ such that for all finite τ -models \mathfrak{A}

$$\mathfrak{A} \models \varphi \Leftrightarrow \mathfrak{A} \models \psi.$$

2. Let τ be finite vocabulary, and \mathfrak{A} and \mathfrak{B} a finite τ -models. Show that the following are equivalent:

1. \mathfrak{A} and \mathfrak{B} satisfy the same sentences of FO^k ,
2. \mathfrak{A} and \mathfrak{B} satisfy the same sentences of $\mathcal{L}_{\infty,\omega}^k$.

Hint: Show that the claim of Exercise 1 holds for the logics $\mathcal{L}_{\infty,\omega}$ and FO when equivalence with respect to all finite models is replaced by equivalence with respect to \mathfrak{A} and \mathfrak{B} .

3. Let $n > 0$. Construct an FO^2 -formula $\psi_n(x)$ such that for any finite linear order \mathfrak{A} the following holds: $\mathfrak{A} \models \psi_n[a/x]$ iff a has exactly n predecessors with respect to $\leq^{\mathfrak{A}}$.

4. Let K be a class of finite ordered τ -models, where $\tau (\leq \in \tau)$ is a finite vocabulary consisting of relation symbols of arity at most 2. Show that K can be axiomatized by a sentence of $\mathcal{L}_{\infty,\omega}^2$.

5. Determine the greatest k such that $\mathbb{G} \cong_k \mathbb{G}'$ holds.

