Finite model theory

Problems 4

Tuesday 2.10.2018

1. Let $\mathfrak{G}$ and $\mathfrak{G}'$ be graphs of cardinality 4. Show that the Spoiler has a winning strategy in the game $\mathrm{EF}_2(\mathfrak{G}, \mathfrak{G}')$

a) if $\mathfrak{G} \not\cong \mathfrak{G}'$ and $\mathfrak{G}$ has exactly 0 or 6 edges.

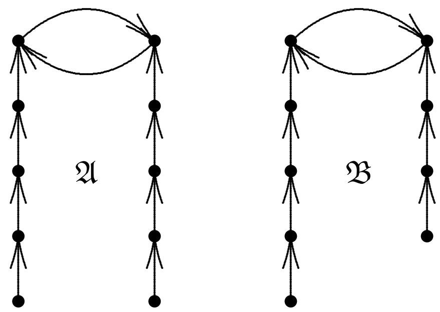
b) if $\mathfrak{G}$ has a node with no edges and $\mathfrak{G}'$ does not have such a node.

2. Let $\mathfrak{A}$, $\mathfrak{B}$, and $\mathfrak{C}$ be τ-models and $k \in \mathbb{N}$. Show that if $\mathfrak{A} \cong_k \mathfrak{B}$ and $\mathfrak{B} \cong_k \mathfrak{C}$, then $\mathfrak{A} \cong_k \mathfrak{C}$.

3. Let $\mathfrak{A}$ and $\mathfrak{B}$ be finite orderings such that $|Dom(\mathfrak{A})| \neq |Dom(\mathfrak{B})|$ and $|Dom(\mathfrak{A})| < 2^k - 1$. Show that the Spoiler has a winning strategy in $\mathrm{EF}_k(\mathfrak{A}, \mathfrak{B})$.

4. Let $\mathfrak{A}$ and $\mathfrak{B}$ be finite orderings such that $|Dom(\mathfrak{A})|, |Dom(\mathfrak{B})| \geq 2^k - 1$. Show that the Duplicator has a winning strategy in $\mathrm{EF}_k(\mathfrak{A}, \mathfrak{B})$.

5. Let R be a binary relation and let $\mathfrak{A}$ and $\mathfrak{B}$ be the following $\{R\}$-models. Determine the largest k such that $\mathfrak{A} \cong^k \mathfrak{B}$.



6. Let $\tau = \{U, V\}$ where U and V are unary relation symbols. Show that there is no τ-sentence φ of first-order logic such that for all finite τ-models $\mathfrak{A}$ holds:

$$\mathfrak{A} \models \varphi \Leftrightarrow |U^{\mathfrak{A}}| = |V^{\mathfrak{A}}|.$$