

Finite model theory

Problems 5

Tuesday 9.10.2018

1. Let \mathfrak{A} and \mathfrak{B} be τ -models, where τ is finite and relational. Furthermore, let $r \in \mathbb{N}$, $\bar{a} \in \text{Dom}(\mathfrak{A})^n$, and $\varphi_{\bar{a}, \mathfrak{A}}^r(\bar{x})$ be the r -Hintikka formula of \bar{a} in \mathfrak{A} . Give a sketch of the proof that $\mathfrak{B} \models \varphi_{\bar{a}, \mathfrak{A}}^r[\bar{b}/\bar{x}]$ iff for all $\psi(\bar{x})$ with $\text{qr}(\psi) \leq r$:

$$\mathfrak{A} \models \psi[\bar{a}/\bar{x}] \Leftrightarrow \mathfrak{B} \models \psi[\bar{b}/\bar{x}].$$

2. Let τ be a finite vocabulary consisting of unary relation symbols. Use Hanf's Theorem to formulate a sufficient condition for $\mathfrak{A} \cong_k \mathfrak{B}$, where \mathfrak{A} and \mathfrak{B} are τ -models and k a natural number.

3. Show using Hanf's Theorem that acyclicity of finite graphs cannot be defined in first-order logic.

4. Show that connectivity of finite graphs cannot be defined $\mathcal{L}_{\infty, \omega}^2$.

5. Construct a sentence of $\mathcal{L}_{\infty, \omega}^3$ that expresses connectivity of finite graphs.