

Finite model theory

Problems 6

Tuesday 16.10.2018

1. Let Σ be a finite and non-empty alphabet and τ_Σ the corresponding vocabulary of Σ word models. Construct a $\text{FO}[\tau_\Sigma]$ -sentence φ_Σ such that for all finite \mathfrak{A} :

$$\mathfrak{A} \models \varphi_\Sigma \iff \mathfrak{A} \cong \mathfrak{A}_w, \text{ for some } w \in \Sigma^+.$$

2. Let $\Sigma = \{a, b, c\}$. Construct finite automata recognizing the following languages:

1. $L_0 = \{w \in \Sigma^+ \mid |w| = 0 \pmod{4}\}$
2. $L_1 = \{w = \alpha_0 \dots \alpha_j \in \Sigma^+ \mid \alpha_i \neq \alpha_{i+1} \text{ for all } 0 \leq i \leq j-1\}$
3. $L_2 = \{w \in \Sigma^+ \mid w = a^k b^l c^t \text{ for some } k, l, t \geq 1\}$

3. Let $L \subseteq \Sigma^*$ be a finite language. Show that L can be recognized by a finite automaton.

4. Show that the languages L_1 and L_2 above can be defined in first-order logic.

5. Show that the language $L_0 \setminus \{\lambda\}$ cannot be defined in first-order logic.

6. Let \mathfrak{A} , \mathfrak{A}' , \mathfrak{B} , and \mathfrak{B}' be finite ordered relational τ -models such that $\mathfrak{A} \cong_k \mathfrak{A}'$ and $\mathfrak{B} \cong_k \mathfrak{B}'$. Show that for the ordered sums the following holds:

$$\mathfrak{A} \uplus \mathfrak{B} \cong_k \mathfrak{A}' \uplus \mathfrak{B}'.$$