# Finite model theory

Problems 7

Tuesday 30.10.2018

**1.** Let $L \subseteq \Sigma^*$ be recognized by a finite automaton. Show that there exists $m \in \mathbb{N}$ such that for all $s \in \Sigma^*$ with $|s| \geq m$, $s$ can be written as $uvw$, where $u, v, w \in \Sigma^*$, $v \neq \lambda$, $|uv| \leq m$, and for all $k \in \mathbb{N}$

$$uvw \in L \Leftrightarrow uv^k w \in L.$$

**2.** Let $\Sigma$ have at least two letters. Show that the language $L$ consisting of all palindromes $w$ over $\Sigma$ cannot be recognized by a finite automaton. Recall that a word $w = \alpha_0 ... \alpha_n$ is a palindrome if $\alpha_i = \alpha_{n-i}$ for all $i$.

**3.** Let $\Sigma = \{a, b\}$, and $L = \{w \in \Sigma^+ \mid w \text{ has more occurrences of } a \text{ than } b\}$. Show that $L$ cannot be recognized by a finite automaton.

**4.** Construct a sentence $\varphi \in \mathrm{MSO}$ such that for all finite graphs $\mathbb{G}$:

$$\mathbb{G} \models \varphi \Leftrightarrow \mathbb{G} \text{ is 3-colorable.}$$

(The class of 3-colorable graphs in not known to in PTIME.)

**5.** Let $L \subseteq \Sigma^+$ be definable in MSO. Show that $L$ is in PTIME.

**6.** Let $\tau = \{P_1, ..., P_l\}$, be a unary vocabulary, and let $\mathfrak{A}$ and $\mathfrak{B}$ be $\tau$-models such that $\mathfrak{A} \cong_{2^k} \mathfrak{B}$. Sketch a proof using the EF-game for MSO that $\mathfrak{A} \equiv_k^{\mathrm{MSO}} \mathfrak{B}$. Use this result to show that for every $\varphi \in \mathrm{MSO}[\tau]$ there is $\varphi^* \in \mathrm{FO}[\tau]$ such that for all finite $\tau$-models $\mathfrak{A}$:

$$\mathfrak{A} \models \varphi \Leftrightarrow \mathfrak{A} \models \varphi^*.$$