HOMEWORK 1

Each problem: 10 points.

On your homework, please write down your **full name.**
For our first set of homework, I think the difficulty is from medium to difficult. If you cannot finish all of them, it is perfectly fine. I intentionally designed this homework to measure the average level of the class. Difficulty of later homework will be adjusted. Again, I want to emphasize, please write down whatever you get, since partial credits will be given.

1. Let A be a Noetherian ring, $I \subset A$ an ideal, and let $B = A/I$ be the quotient ring. Show that:
   (1) B is a Noetherian A-module.
   (2) B is a Noetherian ring.

2. Show that the $\mathbb{Z}$-module $\mathbb{Q}$ is *not* a Noetherian $\mathbb{Z}$-module.

3. Let k be a field, then $k[x]$ is Euclidean domain.

4. Let $\mathbb{Z}[\sqrt{2}]$ be the set of elements $a + b\sqrt{2}$ with $a, b \in \mathbb{Z}$. Show that it is a ring. Show that it is an Euclidean domain.

5. Let $M = \mathbb{Z}e \oplus \mathbb{Z}f$. Let M' be the submodule generated by $2e$ and $3f$. Find a basis of M' as in Part(b) of the Main theorem of "finite free modules over PID". (Samuel BOOK, page 21, Thm 1). You need to show calculations. (not just final answers)

6. Let $A = \mathbb{Z}[i]$. Let $M = Ae \oplus Af$. Let M' be the submodule generated by $2e$ and $(3 + i)f$. Find a basis of M' as in Part(b) of (Samuel BOOK, page 21, Thm 1). You need to show calculations.