

HOMEWORK 3

- (1) (20pts) Let $K = \mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$. Find $x \in K$ such that $K = \mathbb{Q}[x]$ (i.e., x is a primitive element).
- (2) (15pts) Let $A = \mathbb{Z}$, $B = \mathbb{Z}[\sqrt{d}]$ with d a square-free integer.
 - (a) Calculate the norm and trace of $x = \sqrt{d}$.
 - (b) Calculate the norm and trace of $x = d + \sqrt{d}$.
 - (c) Calculate the discriminant of $(\sqrt{d}, d + \sqrt{d})$.
- (3) (10pts) Show that $\frac{\sqrt{5}}{3} + \frac{\sqrt{3}}{2} + \sqrt{2}$ is not integral over \mathbb{Z} .
- (4) (10pts) Prove that $x^5 + 48x + 24$ is irreducible in $\mathbb{Q}[x]$.