Geometric measure theory part II

Exercise set 8

Exercise 1 Let $r > 0$. Let $\{U_i\}_{i=1}^m$ be a collection of disjoint open sets in $\mathbb{R}^d$, so that each U_i contains a ball of radius $a_1 r$ and is contained in a ball of radius $a_2 r$. Then any ball of radius r intersects at most $(1 + 2a_2)^d (a_1)^{-d}$ of the closures of the sets U_i.

Exercise 2 Let $\mathcal{G}$ be a finite graph with vertexes V and edges E. Then it holds that

$$\sum_{v \in V} \deg(v) = 2|E|,$$

where $\deg v = |\{e \in E : e = (v, w) \text{ for some } w \in V\}|$.

Exercise 3 In the proof of Ramsey's theorem, we showed the inequality

$$R(r, s) \leq R(r-1, s) + R(r, s-1).$$

Prove that whenever $R(r-1, s)$ and $R(r, s-1)$ are even, we can improve the above to

$$R(r, s) \leq R(r-1, s) + R(r, s-1) - 1.$$

Exercise 4 Prove that in a party (of finite number of people), there must be an even number of people, that have shaken hands with an odd number of people.

Exercise 5 Prove that in a party of six people, either there are three that don't know each other, or there are three that all know each other. Prove that this does not need to happen in a party of five people.

Exercise 6 Prove that $R(4, 3) \leq 9$. (Hint: first observe that $R(r, 2) = r$ for all $r \in \mathbb{N}$.)