Kaon physics

...or an introduction to flavor oscillations and CP violation

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Autumn 2018

Contents

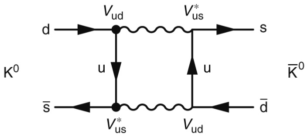
Kaons are the lightest strange mesons and relatively long-lived. They are from the experimental point of view the easiest platform to study many of the interesting features of flavor physics with high precision. In this lecture we shall

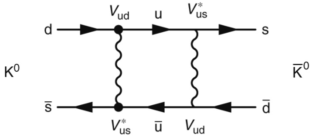
- study the flavor and CP eigenstates of kaons
- study how CP violation emerges in kaon physics
- study the flavor oscillations between K^0 and $\overline{K}^0$

This lecture corresponds to chapters 14.4, 14.5.1, 14.5.2 and 14.5.4 of Thomson's book.

Weak interactions mix the neutral kaon states K^0 and $\overline{K}^0$

- Neutral kaons are produced in two different quark compositions K^0 ($d\bar{s}$) and $\overline{K}^0$ ($\bar{d}s$) — notice that for historical reasons, the strangeness quantum number is so defined that $S(K^0) = +1$, *i.e.* the antiquark has positive strangeness
- As neutral kaons propagate, they can transform between each other through box diagrams, so neither K^0 nor $\overline{K}^0$ is an eigenstate of the full Hamiltonian





The physical eigenstates have differing lifetimes

- Experimentally we observe that there are two different neutral kaon states with (almost) the same mass (close to 500 MeV) but different lifetimes, one having $\tau \simeq 0.1$ ns and the other with $\tau \simeq 50$ ns, these are called K_S and K_L (for short and long)
- We know that in the leptonic sector CP is (at least within current experimental precision) conserved so we may expect the CP eigenstates to be (at least close to) the stationary states
- Kaons are $J^P = 0^-$ mesons so $P|K^0\rangle = -|K^0\rangle$ and $P|\overline{K}^0\rangle = -|\overline{K}^0\rangle$
- The flavor eigenstates have opposite flavor contents so $C|K^0\rangle = e^{i\zeta}|\overline{K}^0\rangle$ and $C|\overline{K}^0\rangle = e^{-i\zeta}|K^0\rangle$, where ζ is an unobservable phase
- If we choose $\zeta = \pi$ (which is a common convention) $C|K^0\rangle = -|\overline{K}^0\rangle$ and $C|\overline{K}^0\rangle = -|K^0\rangle$
- Hence $CP|K^0\rangle = |\overline{K}^0\rangle$ and $CP|\overline{K}^0\rangle = |K^0\rangle$

The CP eigenstates are combinations of K^0 and $\overline{K}^0$

From the CP properties of K^0 and $\overline{K}^0$ it is easy to construct the CP eigenstates:

CP eigenstates of neutral kaons

$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$ with $CP = +1$ and

$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$ with $CP = -1$

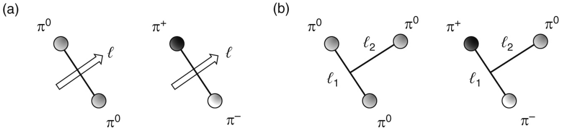
- In weak interactions CP is nearly conserved so these almost coincide with the physical eigenstates K_S and K_L
- On the other hand flavor eigenstates are linear combinations of $|K_1\rangle$ and $|K_2\rangle$ but this combination will have a non-trivial time evolution
- As we shall see, some of the decays are spesific to flavor eigenstates and some to CP eigenstates, which will give us interesting probes on the kaon system

Decays to pions determine the CP properties of K_S and K_L

- Experimentally we see that K_S decays mainly to $\pi^0\pi^0$ or $\pi^+\pi^-$, while K_L decays to $\pi^0\pi^0\pi^0$ or $\pi^+\pi^0\pi^-$
- These decay modes determine the lifetimes — for the three pion decay there is less phase space available so the decay is slower
- As both kaons and pions are $J^P = 0^-$ mesons, the pions must be in an $\ell = 0$ state in the decay $K^0 \to \pi^0\pi^0, \pi^+\pi^-$ to conserve angular momentum
- Hence the parity of the final state is $P(\pi^0\pi^0) = (-1)^\ell P(\pi^0)P(\pi^0) = 1 \cdot (-1) \cdot (-1) = 1$, similarly for $\pi^+\pi^-$
- The neutral pion is a state $|\pi^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ so its C-parity is $+1$ as C transforms the pion to itself
- For $\pi^+\pi^-$ C is equivalent to the exchange of particles and for bosons this does not change the sign
- Hence $\mathrm{CP}(\pi^0\pi^0) = +1$ and $\mathrm{CP}(\pi^+\pi^-) = +1$ implying that K_S could be identified with K_1

Decays to pions determine the CP properties of K_S and K_L

- The angular momentum argument is trickier for the three pion final state



- We first take the angular momentum of two pions labeling it $\vec{L}_1$ and then the angular momentum of the third with respect to the center-of-mass of the pair ($\vec{L}_2$)
- The total angular momentum is $\vec{L} = \vec{L}_1 + \vec{L}_2$, which must be zero due to angular momentum conservation (the spins are all zero), giving $|L_1| = |L_2|$
- Hence the parity is $(-1)^{L_1} \cdot (-1)^{L_2} \cdot (P(\pi^0))^3 = -1$

Decays to pions determine the CP properties of K_S and K_L

- The C-parities of the three-pion final states are $+1$ by the same arguments as for the two pion case
- Hence the CP for the three pion state is negative, which implies $CP(K_L) = -1$
- Thus $|K_L\rangle$ can be identified with $|K_2\rangle$
- In 1964 Christenson, Cronin, Fitch and Turlay observed the decay $K_L \to \pi^+\pi^-$, which was the first (and together with other similar kaon decays, the only for more than 30 years) indication of CP violation in weak interactions (Fitch and Cronin got the 1980 Nobel Prize for this discovery)

Particle decays can be described by complex energy eigenvalues

- We shall first look at kaon state vectors in the limit of CP conservation
- Kaons are produced as flavor eigenstates, *e.g.* $K^0 = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$
- The time evolution can be properly described if we attach an imaginary part to the eigenvalue of the Hamiltonian, *e.g.* $|K_S(t)\rangle = |K_S(0)\rangle e^{-im_S t - \Gamma_S t/2}$
- In such a case $\langle K_S(t)|K_S(t)\rangle = e^{-\Gamma_S t}$ so the wave function dies out in the timescale $\tau_S = 1/\Gamma_S$
- As there is a factor of 500 between the lifetimes of K_S and K_L, after a reasonably long propagation time a beam initially in a flavor eigenstate has become a pure $|K_L\rangle$ state

CP is nearly conserved — only a tiny fraction of decays are CP violating

K_S decays	BR	K_L decays	BR
$K_S \to \pi^+\pi^-$	69.2%	$K_L \to \pi^+\pi^-$	0.20%
$K_S \to \pi^0\pi^0$	30.7%	$K_L \to \pi^0\pi^0$	0.09%
$K_S \to \pi^0\pi^0\pi^0$	$< 2.6 \times 10^{-8}$	$K_L \to \pi^0\pi^0\pi^0$	19.5%
$K_S \to \pi^+\pi^-\pi^0$	3.5×10^{-7}	$K_L \to \pi^+\pi^-\pi^0$	12.5%
$K_S \to \pi^- e^+ \nu_e$	0.03%	$K_L \to \pi^- e^+ \nu_e$	20.3%
$K_S \to \pi^+ e^- \overline{\nu}_e$	0.03%	$K_L \to \pi^+ e^- \overline{\nu}_e$	20.3%

- Here the leptonic modes have equal decay rates but due to the larger total rate of K_S, the branching ratios are smaller
- There is a clear difference in the order of magnitude of the CP violating decays, 2 orders of magnitude when kinematics are favorable and 6 orders of magnitude when they are not

CP is violated both in kaon mixing and decays

- There are two possible sources for CP violation: The Hamiltonian can introduce CP violation in the kaon mixing so that the physical eigenstates do not coincide with the CP ones, it could also be possible for the kaon decays to be directly affected by the CP violating phase
- In the first case the eigenstates are parametrized as

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1\rangle + \epsilon|K_2\rangle), \quad |K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle + \epsilon|K_1\rangle),$$

  where ϵ is a small complex parameter
- The second case is parametrized by $\epsilon' = \Gamma(K_2 \to \pi\pi)/\Gamma(K_2 \to \pi\pi\pi)$
- Both are nonzero and small and $\Re(\epsilon'/\epsilon) \simeq 1.65 \times 10^{-3}$

The Hamiltonian gets contributions from interaction potentials and decays

We shall next go through the kaon mixing and see how most of the CP violation is generated. The mixing is a second order effect in weak interactions, *i.e.* proportional to G_F^2.

- The Hamiltonian equation of motion for, say, $|K^0\rangle$ in its rest frame is $i\frac{\partial}{\partial t}|K^0(t)\rangle = (m - \frac{i}{2}\Gamma)|K^0(t)\rangle$
- The mass term is the sum of the quark masses and the potential energy of the interactions between quarks:

$$m = m_d + m_s + \langle K^0|H_{QCD} + H_{QED} + H_W|K^0\rangle + \sum_j \frac{\langle K^0|H_W|j\rangle\langle j|H_W|K^0\rangle}{E_j - m_K}$$

- The decay rate is determined by the Fermi golden rule: $\Gamma = 2\pi \sum_f |\langle f|H_W|K^0\rangle|^2 \rho_f$, where the sum goes over all possible final states $|f\rangle$ and ρ_f is the corresponding density of states

Weak interactions introduce mixing terms to the Hamiltonian

- To introduce K^0–$\overline{K}^0$ mixing, we have to generalize the Hamiltonian to a matrix
- We parametrize the state as $|K(t)\rangle = a(t)|K^0\rangle + b(t)|\overline{K}^0\rangle$
- The equation of motion is generalized to

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t)|K^0\rangle \\ b(t)|\overline{K}^0\rangle \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} \begin{pmatrix} a(t)|K^0\rangle \\ b(t)|\overline{K}^0\rangle \end{pmatrix}$$

- The mass terms M_{11} and M_{22} are similar to the ones before, but M_{12} and M_{21} arise from the box diagrams:

$$M_{12} = M_{21}^* = \sum_j \frac{\langle\overline{K}^0|H_W|j\rangle\langle j|H_W|K^0\rangle}{E_j - m_K}$$

Weak interactions introduce mixing terms to the Hamiltonian

The decay term can be written as

$$\sum_f |\langle f|H_W|K(t)\rangle|^2 = \sum_f |\langle f|H_W|(a(t)|K^0\rangle + b(t)|\overline{K}^0\rangle)|^2 =$$

$$\sum_f |a(t)|^2\langle K^0|H_W|f\rangle\langle f|H_W|K^0\rangle + |b(t)|^2\langle\overline{K}^0|H_W|f\rangle\langle f|H_W|\overline{K}^0\rangle +$$

$$a^*(t)b(t)\langle\overline{K}^0|H_W|f\rangle\langle f|H_W|K^0\rangle + a(t)b^*(t)\langle K^0|H_W|f\rangle\langle f|H_W|\overline{K}^0\rangle$$

The first two terms can be identified as Γ_{11} and Γ_{22}, whereas the two latter ones are Γ_{12} and $\Gamma_{21} = \Gamma_{12}^*$.

Diagonalizing the Hamiltonian leads to the physical eigenstates

- From the definition we see that M_{11}, M_{22}, Γ_{11} and Γ_{22} are real, whereas M_{12} and Γ_{12} are complex (and are the source of CP violation)
- CPT invariance requires $M_{11} = M_{22} = M$ and $\Gamma_{11} = \Gamma_{22} = \Gamma$ so the equation of motion simplifies to

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t)|K^0\rangle \\ b(t)|\overline{K}^0\rangle \end{pmatrix} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} a(t)|K^0\rangle \\ b(t)|\overline{K}^0\rangle \end{pmatrix}$$

- We may solve the eigenvalues of the Hamiltonian giving

$$E_\pm = M - \frac{i}{2}\Gamma \pm \sqrt{\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)},$$

  the real parts giving the masses and the imaginary parts giving the decay widths

The eigenstates differ slightly from $|K_{1,2}\rangle$

- The eigenvectors of the Hamiltonian (*i.e.* the stationary states) can be solved and they are $\frac{1}{\sqrt{1+|\xi|^2}}(|K^0\rangle + \xi|\overline{K}^0\rangle$ and $\frac{1}{\sqrt{1+|\xi|^2}}(|K^0\rangle - \xi|\overline{K}^0\rangle$, where

$$\xi = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right)^{1/2}$$

- If M_{12} and Γ_{12} were real, $\xi = 1$ and the eigenstates would coincide with $|K_{1,2}\rangle$, however they are complex, which leads to CP violation
- Rewriting $\xi = \frac{1-\epsilon}{1+\epsilon}$ leads to eigenstates

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1\rangle + \epsilon|K_2\rangle)$$

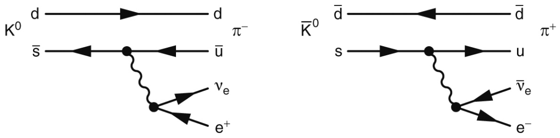
$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle + \epsilon|K_1\rangle)$$

The kaon eigenstates have a tiny mass difference

- From the eigenvalues we get
  $E_+ - E_- = 2\sqrt{\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)}$
- We may write $E_\pm = M \pm \Delta M/2 - \frac{i}{2}(\Gamma \pm \Delta\Gamma/2)$, where $\Delta M = |\Re(E_+ - E_-)|$ and $\Delta\Gamma = \pm|2\Im(E_+ - E_-)|$, where the sign of $\Delta\Gamma$ depends on the experimental data on the lifetimes
- Experimentally the heavier state has the longer lifetime, so it is associated with $|K_L\rangle$
- The states K_S and K_L have a tiny mass difference $\Delta m_K = 3.5 \times 10^{-15}$ GeV (we will see soon, how can this be measured — the masses are not known to this precision)

The kaons oscillate between K^0 and $\overline{K}^0$ and relax to a state with equal weights

- Kaons are produced in flavor eigenstates, but since they are not the eigenstates of the Hamiltonian, they have a nontrivial time evolution, which leads to flavor oscillations
- The oscillations are due to the box diagrams with two W-bosons
- Since the K_S component decays faster, the oscillations will eventually end as the K_L has fixed proportions of K^0 and $\overline{K}^0$
- Measuring flavor oscillations is possible because the semileptonic decays show the flavor of the kaon: Only $K^0 \to \pi^- e^+ \nu_e$ and $\overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$ are possible, so the charge of the electron (or pion) tells us, which flavor the kaon had



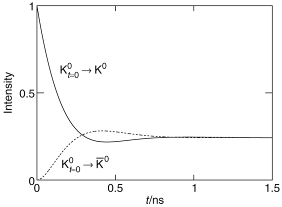
The time evolution is dictated by the CP eigenstates

We'll first neglect CP violation as it is a small effect. Look at a state produced as K^0:

- The initial state is $|K^0\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$
- This evolves in time as $|K(t)\rangle = \frac{1}{\sqrt{2}}(\theta_S|K_S\rangle + \theta_L|K_L\rangle) = \frac{1}{2}(\theta_S + \theta_L)|K^0\rangle + \frac{1}{2}(\theta_S - \theta_L)|\overline{K}^0\rangle$
- Here $\theta_{S,L} = e^{-im_{S,L}t - \Gamma_{S,L}t/2}$ are the phase factors for decaying eigenstates of the Hamiltonian
- The probability of observing a kaon as K^0 is $P(K^0_{t=0} \to K^0_t) = |\langle K^0|K(t)\rangle|^2 = \frac{1}{4}|\theta_S + \theta_L|^2$
- Similarly the probability of observing a $\overline{K}^0$ is $P(K^0_{t=0} \to \overline{K}^0_t) = |\langle \overline{K}^0|K(t)\rangle|^2 = \frac{1}{4}|\theta_S - \theta_L|^2$

The oscillation probability is sensitive to the mass difference

- Expanding these gives $|\theta_S \pm \theta_L|^2 = \underbrace{|\theta_S|^2}_{=e^{-\Gamma_S t}} + \underbrace{|\theta_L|^2}_{=e^{-\Gamma_L t}} \pm 2\Re(\theta_S \theta_L^*)$
- The last term is $2\Re(e^{-(\Gamma_S+\Gamma_L)t/2 - i\Delta m_K t}) = 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m_K t)$
- Hence the oscillation period is sensitive to the kaon mass difference (like neutrino oscillations), although the amplitude is decaying rapidly
- The mass difference is so small that the oscillation period is longer than the K_S lifetime so only the first oscillation is somewhat visible



The CPLEAR experiment measured kaon flavor oscillations

Kaons were produced by $p\bar{p} \to \pi^+ K^- K^0$, $\pi^- K^+ \overline{K}^0$, the kaon charge gives the neutral kaon flavor

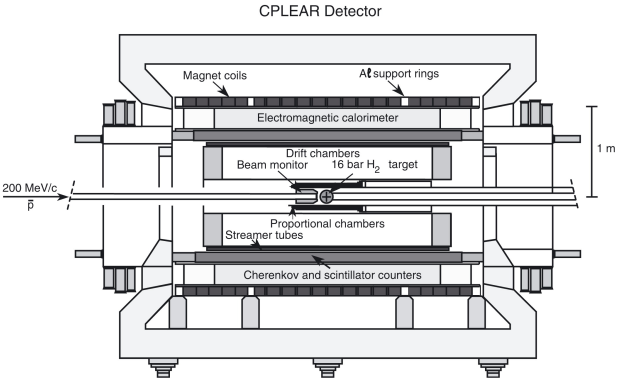
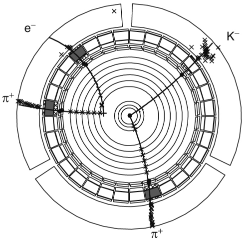


Figure: The CPLEAR collaboration, Phys. Rept. 403 (2004) 303

Cerenkov counters were used to identify kaons



- The kinematics were such that pions, electrons and muons gave a Cerenkov signal, while kaons did not — helps in triggering and particle identification
- The low energy meant that the neutral kaons decayed within the detector, their flight distance indicating the lifetime
- The events of interest were the semileptonic decays of kaons, where the charge of the lepton gave the flavor of the kaon

Many uncertainties cancel when the relative difference in rates is considered

- The probability of K^0 decaying as a K^0 is measured from events with a K^- and e^+ in the final state, this will be proportional to $e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m_K t)$
- The probability of K^0 oscillating to a $\overline{K}^0$ before decaying is measured from events with a K^- and e^- in the final state, which is proportional to $e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m_K t)$
- When considering the ratio

$$A(t) = \frac{P(K^0_{t=0} \to K^0_t) - P(K^0_{t=0} \to \overline{K}^0_t)}{P(K^0_{t=0} \to K^0_t) + P(K^0_{t=0} \to \overline{K}^0_t)}$$

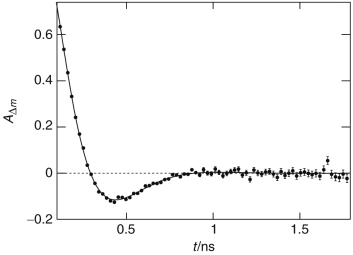
uncertainties related to the production rate cancel, it is possible to include also the similar rates for $\overline{K}^0$

The asymmetry in the decays allows us to measure Δm_K

- Plugging in the expressions for the various rates leads to the expression

$$A(t) = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m_K t)}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

  for the asymmetry in the kaon decays
- The measured asymmetry is consistent with $\Delta m_K = 3.5 \times 10^{-15}$ GeV, whereas the masses themselves have been measured with a precision of 10^{-5} GeV



CP violation can be seen in semileptonic decays of K_L

- We may observe CP violation in the semileptonic decays if we look at events far from the interaction point, where the state is a pure $|K_L\rangle$ state
- Since $|K_L\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\overline{K}^0\rangle]$, there will be a slight difference in the rates of the semileptonic decays
- We have $\Gamma(K_L \to \pi^{\pm} e^{\mp} \nu) \propto |1 \mp \epsilon|^2 \simeq 1 \mp 2\Re(\epsilon)$
- This allows the measurement of CP violation in terms of the asymmetry parameter (writing $\epsilon = |\epsilon| e^{i\phi}$)

$$\delta = \frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \overline{\nu}_e)}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \overline{\nu}_e)} \simeq 2\Re(\epsilon) = 2|\epsilon| \cos\phi$$

- Experimentally we have $\delta = 0.327 \pm 0.012\%$ showing again that CP is violated in weak interactions ($|\epsilon| \gtrsim 1.6 \times 10^{-3}$)

Next time: How to measure the full ϵ and not just its real part

Summary

- Kaons are usually produced as flavor eigenstates, they propagate nearly as CP eigenstates (modulo small CP violation) and they may decay as either of them
- Most of the CP violation in the neutral kaon system comes from the K^0–$\overline{K}^0$ mixing through the box diagrams
- The flavor oscillations can be seen in semileptonic decays and they are sensitive to the tiny mass difference between K_S and K_L
- Also CP violation can be seen in the semileptonic decays of kaons