

HOMEWORK 5

- (1) Let (20pts) $K = \mathbb{Q}(\sqrt{-7})$. A : ring of integers of K .
 - (a) Find a \mathbb{Z} -basis of A . (hint: use Theorem 1 on page 35 of Book)
 - (b) Calculate the absolute discriminant d .
 - (c) Calculate the norm bound (round to two decimal places) as in Corollary 1 on page 58 of BOOK.
 - (d) Conclude that A is PID.
- (2) (20pts) $K = \mathbb{Q}(\sqrt{-5})$. A : ring of integers of K .
 - (a) Calculate the absolute discriminant d .
 - (b) Calculate the norm bound (round to two decimal places) as in Corollary 1 on page 58 of BOOK.
 - (c) Show that the norm of the ideal $\alpha = (2, 1 + \sqrt{-5})$ is 2. Show that it is the *unique* ideal in A of norm 2.
 - (d) Conclude that the class number $\text{card}(C(A)) = 2$. (Hint: first show that A is not PID, then show its class number is at most 2.)