

Finite model theory
Problems 12
Tuesday 4.12.2018

1. Let $\tau = \{R\}$, where $ar(R) = 2$. Show that $l(K) = 1$, where

$$K = \{\mathfrak{A} \mid \mathfrak{A} \text{ is finite connected } \tau\text{-model}\}.$$

We view τ -models \mathfrak{A} as directed graphs and \mathfrak{A} is said to be connected if there is a directed path between any two vertices of \mathfrak{A} .

2. Let $\tau_1 = \{\leq\}$. By extending the definitions of $l_n(\varphi)$ and $l(\varphi)$ in the obvious way for ordered models, show that FO has a 0-1 law for τ_1 -models.
3. Let $\tau_2 = \{\leq, P\}$ where P is unary. Show that FO does not satisfy a 0-1 law for ordered τ_2 -models.
4. Let τ be finite and relational vocabulary and \mathfrak{A} a finite τ -model. Define K by $K := \{\mathfrak{A}' \mid \mathfrak{A} \text{ embeds to } \mathfrak{A}'\}$. Show that $l(K) = 1$. (Hint: Use the extension axioms.)
5. Let τ be finite and relational and let T_{rand} be the set of all r -extension axioms for all $r \in \mathbb{N}$. Assume that \mathfrak{A} and \mathfrak{B} are countably (infinite) models of T_{rand} . Sketch a proof that $\mathfrak{A} \cong \mathfrak{B}$. (Hint: construct an isomorphism between the models using the extension axioms.)