Finite model theory

Problems 13

Tuesday 11.12.2018

1. Let τ be a finite and relational vocabulary and $k \in \mathbb{N}$. Suppose $\mathfrak{A}$ and $\mathfrak{B}$ are finite τ-models.

a) Give the definition of $\mathfrak{A}$ and $\mathfrak{B}$ being partially isomorphic up to k ($\mathfrak{A} \cong_k \mathfrak{B}$)?

b) Give an example of a pair of non-isomorphic graphs $\mathbb{G}$ and $\mathbb{G}'$ satisfying $\mathbb{G} \cong_2 \mathbb{G}'$ (show this using the definition of $\cong_k$).

2. Let $\tau = \{U, V\}$ where U and V are unary relation symbols. Show that there is no τ-sentence φ of $\mathcal{L}^{\omega}_{\infty,\omega}$ such that for all finite τ-models $\mathfrak{A}$ holds:

$$\mathfrak{A} \models \varphi \Leftrightarrow |U^{\mathfrak{A}}| = |V^{\mathfrak{A}}|.$$

3. When is a sentence of the form $[\mathrm{IFP}_{\overline{x},R}\varphi]\overline{t}$ true in a model $\mathfrak{A}$? Construct a sentence φ of FO(IFP) such that for all finite graphs $\mathbb{G}$:

$$\mathbb{G} \models \varphi \Leftrightarrow \mathbb{G} \text{ is connected.}$$

4. Construct a sentence φ of monadic second-order logic such that for all finite *ordered* models $\mathfrak{A}$:

$$\mathfrak{A} \models \varphi \Leftrightarrow |\mathrm{Dom}(\mathfrak{A})| \text{ is divisible by three.}$$

5. Let $\Sigma = \{\alpha_1, ..., \alpha_n\}$ be a finite alphabet and $w \in \Sigma^+$.

a) Define the word model $\mathfrak{A}_w$ corresponding to w.

b) Construct a finite automaton that accepts $w \in \{1\}^*$ iff the length of w is divisible by three.