15. Consider a situation in which individuals can die but not reproduce and where the population is maintained by immigration. Assume that death and immigration are independent Poisson processes with a per capita death rate equal to \( \delta \) and a total immigration rate equal to \( \alpha \Omega^p \) where \( \Omega \) is the system size and \( 0 \leq p \leq 1 \). Let further \( N(t) \) be the number of individuals present at time \( t \geq 0 \), and let \( P_n(t) \) denote the probability that \( N(t) = n \) for any \( n \in \{0, 1, 2, \ldots\} \).

(a) Give a system of differential equations for the probabilities \( P_n \).

(b) Show that the stationary distribution is the Poisson distribution. Give the mean and variance of the stationary distribution.

(c) Give an interpretation of \( p = 0, \frac{1}{2}, 1 \) in terms of properties of the real system.

16. (a) Give the semi-large system approximation of the model in (15) in the form of a Fokker-Planck equation (FPE) and in the form of a stochastic differential equation (SDE) for changes in population density.

(b) Use the SDE to compute the mean and variance of the population density at the stationary distribution, and compare the results with those found in (15).

(c) Use the FPE to find the stationary distribution and compare the result with the stationary distribution found in (15).

(d) Show that as system size increases, then the variance of the stationary distribution decreases. Give an Ornstein-Uhlenbeck approximation of the SDE for small perturbations of the population density from the mean density, and calculate the auto-covariance function.

(e) Discuss the role of the parameter \( p \) in all this.