COURSE JOURNAL OF COMPLEX ANALYSIS I /RHS

17.1.2019

Welcome to the course.
Overview of the course.
Practical matters.
Recall the real vector space $(\mathbb{R}^2, +)$ equipped with addtion $+$ and scalar multiplication. Also, recall geometric meaning of addition and subtraction of vectors.
Define multiplication $\cdot : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$.
$(\mathbb{R}^2, +.\cdot)$ is a field.
$(\mathbb{R}^2, +.\cdot) =: \mathbb{C}$ is called the complex plane.
Elements of the complex plane are called complex numbers.

18.1.2019

Cartesian form for a complex number.
Definitions of the real part of z, the imaginary part of z, the complex conjugate of z, the modulus of z.
Basic complex operations for z and its complex conjugate and modulus. The triangle inequality.
Polar form, that is, the the argument-modulus-expression for z.
Euler's formula as a notation.
Geometric meaning of the multiplication of two complex numbers.

24.1.2019

More about the argument.
Applications of De Moivre's theorem.
The n^{th} roots of complex numbers.
About the history of complex numbers and complex analysis.
We started 'Sets in the complex plane'.

Date: May 3, 2019.
Key words and phrases. Complex numbers, Complex functions, Analytic functions.

25.1.2019

Sets in the complex plane.
Limits and continuity.
Cauchy sequences. The complex plane $\mathbb{C}$ is complete.

31.1.2019

We started Section 4: Analytic functions.
Definitions, Examples.
Elementary properties, Differentiation formulas, Examples.

1.2.2019

We started to study the relationship between the complex derivative of a complex valued function and the real derivative of the corresponding real valued function of two real variables.
Starting with an arbitrary analytic function we deduced the Cauchy-Riemann equations.
We studied the relationship between the complex and real derivatives by defining two differential operators $\partial/\partial z$ and $\partial/\partial \bar{z}$.
We proved that a one complex variable, complex valued function is analytic by assuming the real differentiablity of the corresponding mapping $\mathbb{R}^2 \to \mathbb{R}^2$ and the validity of Cauchy-Riemann equations.

Have a nice weekend by enjoying Finnish weather!

7.2.2019

We continued with harmonic functions; starting with examples and then stating and proving two theorems.
We recalled useful results from the Vector Calculus course related to differentiability of real valued vector functions.

8.2.2018

We constructed a function $f = u + iv$ such that the partial derivatives of the real and imaginary parts of f obey the C-R equations at a given point z_0, but the function is not real differentiable at z_0. This example function points out the importance of all the parts in the characterization of complex differentiability at z_0.
The function f is complex differentiable at z_0 if and oly if $L_f(z_0)$ is not only $\mathbb{R}$-linear, but also $\mathbb{C}$-linear.

I gave an overview of the rest of the course.

14.2.2019

Basic properties of complex series: absolute convergence, the n^{th} term test, comparison test, ratio test, root test, some examples (especially, the geometric series).

15.2.2019

Power series. Abel's theorem. A theorem which states that there always exists an open disc (possible empty) on which the power series converges absolutely. Hadamard's formula. A power series is infinitely complex differentiable in the disc of convergence. The derivatives and higher derivatives are also power series and are obtained by termwise differentiation.

Walkways in Helsinki area are very, very slippery now! Just take care and be very careful when walking.

21.2.2019

Elementary algebraic properties and basic mapping properties of the complex exponential function.

22.2.2019

The complex logarithm.

28.2.2019

Curves and contours.
Definite integral of a complex-valued function f over an interval.

1.3.2019

Integration along curves.
Basic properties of integrals along curves.
Antiderivatives, Fundamental theorem of calculus in complex analysis, and its corollary. Examples.

Have a nice break!

21.3.2019

Fundamental theorem of calculus II in complex analysis. Examples.
Goursat's theorem for a rectangle.

22.3.2019

Local existence of antiderivatives. The Cauchy-Goursat theorem in a disc. Cauchy's local integral formula. Examples.

Have a nice weekend!

28.3.2019

Corollaries of the Cauchy formulas: Morera's theorem, Cauchy's inequalities, the power series expansion for analytic functions, Liouville's theorem.

4.4.2019

Analytic continuation. A local study of zeros of an analytic function. Maximum modulus principle.

5.4.2019

The Schwarz lemma. Minimum modulus theorem. On cycles. Winding numbers. The Cauchy global integral formula.

11.4.2019

Corollaries of the Cauchy global integral formula. Deformation theorem. Examples. Integrating rational functions on the real line.

12.4.2019

Integrating trigonometric-rational expressions on the real line. The extended complex plane.

25.4.2019

Möbius transformations.

26.4.2019

Continuing on Möbius transformations. On conformality.

2.5.2019

Review.

The course is over.

The exam will be on Monday, 6th of May starting at 10 (for 3 hours) in the class room B120.

Thank you very much for participating in the course.