MAST31213 Complexity theory
Master's Programme in Mathematics and Statistics
Fall 2019
Exercise set 2

Read chapters 1.1–1.3 of the book.

Exercise 1. Describe a 1-tape Turing machine that adds 1 to a binary number given as input.

Exercise 2. Describe Turing machines accepting each of the following languages, as subsets of $\{0,1,c\}^*$:

(a) $\{0,1,c\}^*$
(b) $\emptyset$
(c) $\{\sigma \in \{0,1,c\}^* \mid \sigma \text{ contains exactly one } c\}$
(d) $\{011\}$
(e) $\{\sigma c^n \sigma \mid \sigma \in \{0,1\}^*, n > 0\}$.

In each case give the set of rules of the Turing machine and a brief description of how it is supposed to work.

Exercise 3. Describe a 1-tape Turing machine that solves the PALINDROME problem (over the alphabet $\{0,1\}$), i.e., calculates the Boolean function $PAL(x)$, where $PAL(x) = 1$ iff x is a palindrome. How many moves does the machine require for an input of length n?

Exercise 4. Design a binary encoding of graphs and a Turing machine such that given a graph on $[n]$ and a vertex k (in binary) the machine returns the degree (i.e. number of neighbours) of k (in binary). You may use a machine with several tapes. What is the length of your code for the complete graph on n vertices? How long does it take to compute the degree of one of its vertices?

Exercise 5. See the definition of *time constructible* function on p. 16 in the book. Give an example of a function that is *not* time constructible.