Dependence Logic

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Lecturer: Fan Yang

Recap

Dependence atom: $=\!(t_1, \ldots, t_n, t')$
Inclusion atom: $t_1 \ldots t_n \subseteq t'_1 \ldots t'_n$
Exclusion atom: $t_1 \ldots t_n \mid t'_1 \ldots t'_n$
Independence atom: $t_1 \ldots t_n \perp t'_1 \ldots t'_m$

Team semantics:

- A team X of $\mathcal{M}$ is a set of assignments $s : V \to \mathrm{dom}(\mathcal{M})$
- Satisfaction relation: $\mathcal{M} \models_X \phi$

	x_1	x_2	$\ldots$	x_n
s_1	a_{11}	a_{12}	$\ldots$	a_{1n}
s_2	a_{21}	a_{22}	$\ldots$	a_{2n}
$\vdots$				
s_m	a_{m1}	a_{m2}	$\ldots$	a_{mn}

Recap and connection to database theory

Dependence atoms:

$$\mathcal{M} \models_X =\!(\vec{t}, t') \quad \text{iff} \quad \text{for all } s, s' \in X,$$
$$s(\vec{t}^{\mathcal{M}}) = s'(\vec{t}^{\mathcal{M}}) \Longrightarrow s(t'^{\mathcal{M}}) = s'(t'^{\mathcal{M}}).$$

$\mathcal{M} \models_X =\!(\vec{x}, y)$

$\exists f$

x_1	x_2	x_3	y	z
a_1	a_2	a_3	b	e
a_1	a_2	a_3	b	a
c_1	c_2	c_3	d	c
c_1	c_2	c_3	d	d

A team – a relational database
Dependence atoms – functional dependencies in database theory

Inclusion atoms – inclusion dependencies
Exclusion atoms – exclusion dependencies in database theory

- $\mathcal{M} \models_X \vec{t_1} \subseteq \vec{t_2}$ iff for all $s \in X$, there exists $s' \in X$ such that $$s(\vec{t_1}^{\mathcal{M}}) = s'(\vec{t_2}^{\mathcal{M}}).$$
- $\mathcal{M} \models_X \vec{t_1} \mid \vec{t_2}$ iff for all $s, s' \in X$, $$s(\vec{t_1}^{\mathcal{M}}) \neq s'(\vec{t_2}^{\mathcal{M}}).$$

$x \subseteq y$

x	y
3	1
5	3
7	5
11	7
3	9
5	11

$x \mid y$

x	y
1	2
3	4
5	6
7	8
9	10

Independence atoms

- $\mathcal{M} \models_X \vec{t}_1 \perp \vec{t}_2$ iff for all $s, s' \in X$, there exists $s'' \in X$ such that $s''(\vec{t}_1^{\mathcal{M}}) = s(\vec{t}_1^{\mathcal{M}})$ and $s''(\vec{t}_2^{\mathcal{M}}) = s'(\vec{t}_2^{\mathcal{M}})$.
- $\mathcal{M} \models_X \vec{t}_1 \perp_{\vec{t}_0} \vec{t}_2$ iff for all $s, s' \in X$ s.t. $s(\vec{t}_0^{\mathcal{M}}) = s'(\vec{t}_0^{\mathcal{M}})$, there exists $s'' \in X$ s.t. $s''(\vec{t}_0^{\mathcal{M}}) = s(\vec{t}_0^{\mathcal{M}})$, $s''(\vec{t}_1^{\mathcal{M}}) = s(\vec{t}_1^{\mathcal{M}})$ and $s''(\vec{t}_2^{\mathcal{M}}) = s'(\vec{t}_2^{\mathcal{M}})$.

$x \perp y$

x	y	z
a	b	e
c	d	d
c	b	e
a	d	a

$x \perp_z y$

x	y	z
a	b	e
c	d	e
c	b	e
a	d	e

– embedded multivalued dependencies in database theory