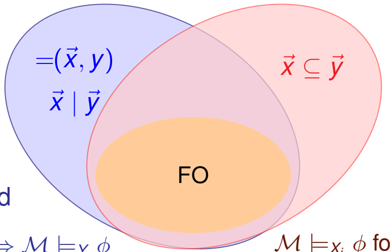
Dependence Logic

September 17 (Tuesday), 2019

$\vec{y} \perp_{\vec{x}} \vec{z}$



Downwards closed

$\mathcal{M} \models_X \phi$ & $Y \subseteq X \Longrightarrow \mathcal{M} \models_Y \phi$

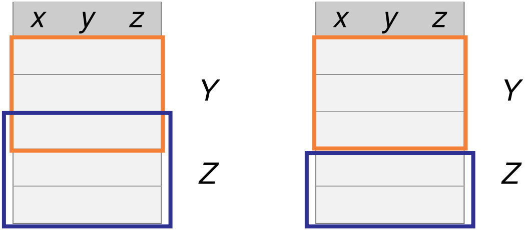
Union closed

$\mathcal{M} \models_{X_i} \phi$ for all $i \in I \Longrightarrow \mathcal{M} \models_{\bigcup_{i \in I} X_i} \phi$

Flat

$\mathcal{M} \models_X \phi \iff \mathcal{M} \models_{\{s\}} \phi$ for all $s \in X$

- $\mathcal{M} \models_X \phi \vee \psi$ iff $\exists Y, Z \subseteq X$ s.t. $X = Y \cup Z$,
  $M \models_Y \phi$ and $M \models_Z \psi$.
- $\mathcal{M} \models^s_X \phi \vee \psi$ iff $\exists Y, Z \subseteq X$ s.t. $X = Y \cup Z$, $Y \cap Z = \emptyset$,
  $M \models_Y \phi$ and $M \models_Z \psi$.



lax semantics strict semantics

Locality: For any two teams X, Y with $X \upharpoonright \mathrm{Fv}(\phi) = Y \upharpoonright \mathrm{Fv}(\phi)$,

$$\mathcal{M} \models_X \phi \iff \mathcal{M} \models_Y \phi.$$

Note: Locality fails for FO($\subseteq$) with strict semantics.

Existential quantifier

- $\mathcal{M} \models_X \exists x\phi$ iff $\mathcal{M} \models_{X(F/x)} \phi$ for some $F : X \to \wp^+(M)$, where
$$X(F/x) = \{s(a/x) \mid s \in X \text{ and } a \in F(s)\}.$$
- $\mathcal{M} \models^s_X \exists x\phi$ iff $\mathcal{M} \models_{X(F_0/x)} \phi$ for some $F_0 : X \to M$, where
$$X(F_0/x) = \{s(F_0(s)/x) \mid s \in X\}.$$

u	v	x
a	b	a
b	c	b
b	d	c
c	a	d
$\vdots$	$\vdots$	$\vdots$

M

lax semantics

u	v	x
a	b	a
b	c	b
b	d	c
c	a	d
$\vdots$	$\vdots$	$\vdots$

M

strict semantics

Locality: For any two teams X, Y with $X \upharpoonright \mathrm{Fv}(\phi) = Y \upharpoonright \mathrm{Fv}(\phi)$,
$$\mathcal{M} \models_X \phi \iff \mathcal{M} \models_Y \phi.$$
Note: Locality fails for FO($\subseteq$) with strict semantics.