MAST31213 Complexity theory
Master's Programme in Mathematics and Statistics
Fall 2019
Exercise set 6

Read chapters 2.3 – 2.4 of the book.

Exercise 1. `VERTEX COVER` is the problem, where given an undirected graph G and an integer k we should decide whether there is a set S of at most k vertices in G such that all edges of the graph has at least one end point in S. Show that `VERTEX COVER` is **NP**-complete.

Exercise 2. Show that `2SAT` is in **P**.

Exercise 3. A reduction f from an **NP**-language L to an **NP**-language L' is *parsimonious* if the number of certificates of x is equal to the number of certificates of $f(x)$. Give a parsimonious polynomial time Karp reduction from `SAT` to `3SAT`. (Hint: When does the reduction we looked at not give a unique satisfying assignment for the 3CNF formula, and what can be done about it without actually checking truth values?)

A language L is polynomial-time Cook reducible *to a language L' if there is a polynomial time Turing machine M that, given an* oracle *for deciding L', can decide L. An oracle for L' is an extra device plugged to M that checks whether $x \in L'$ in one step (e.g. M has an extra tape, s.t. when M writes a string x on this tape and goes into a special "invocation" state, then the string gets overwritten by 1 or 0 depending on whether $x \in L'$ or not, and the overwriting happens in one single step).*

Exercise 4. Show that

(a) the notion of Cook reducibility is transitive, and
(b) `3SAT` is Cook-reducible to `TAUTOLOGY`, where `TAUTOLOGY` is the set of (codes of) Boolean formulas that are satisfied by all assignments.

Exercise 5. Show that $\overline{\texttt{SAT}}$ (i.e. the complement of `SAT`) is **NP**-hard under Cook reductions, i.e. every language in **NP** reduces to $\overline{\texttt{SAT}}$ via a Cook reduction.