

MAST31213 Complexity theory
Master's Programme in Mathematics and Statistics
Fall 2019
Exercise set 7

Read chapters 2.4 – 2.6 of the book.

Exercise 1. Show that there is a nondeterministic universal TM. That is, prove that there exists a representation scheme of NDTMs and a NDTM \mathcal{NU} such that for every string α , and input x , $\mathcal{NU}(x, \alpha) = M_\alpha(x)$. Further show that such a universal NDTM can be made to run only linearly slower than the original machine (the linear coefficient depends on the simulated machine M_α , but not on x).

Exercise 2. Show that **coNP** is closed under union and intersection, i.e., if $L_1, L_2 \in \mathbf{coNP}$ then so are $L_1 \cup L_2$ and $L_1 \cap L_2$.

Exercise 3. Show that **NP=coNP** if and only if **3SAT** and **TAUTOLOGY** are polynomial-time reducible to one another.

Exercise 4. Suppose $L_1, L_2 \in \mathbf{NP} \cap \mathbf{coNP}$. Then show that $L_1 \oplus L_2$ is in $\mathbf{NP} \cap \mathbf{coNP}$ where $L_1 \oplus L_2 = \{x : x \text{ is in exactly one of } L_1, L_2\}$.

Exercise 5. Let R be a relation that is *polynomially bounded*, i.e., there exists a polynomial p such that if $(x, w) \in R$ then $|w| \leq p(|x|)$, and *polynomially verifiable*, i.e. $R \in \mathbf{P}$. Then R defines the **NP**-language

$$L_R := \{x \mid \exists w : (x, w) \in R\}.$$

The *search problem* over R is: given x , find a w for which $(x, w) \in R$, if such a w exists. The *decision problem* over R is: given x , determine whether $x \in L_R$. A relation is *self-reducible* if the search problem over R is Cook-reducible to the decision problem over R . (Cook-reductions were defined in last week's exercises.)

Show that all (polynomially bounded and polynomially verifiable) relations whose decision problems are **NP**-complete, are self-reducible.

(It is conjectured that not all languages in **NP** are self-reducible. An example might be the natural relation corresponding to the factoring problem: deciding whether a number is composite is in **P**, but factoring composite numbers in polynomial time is not believed possible.)